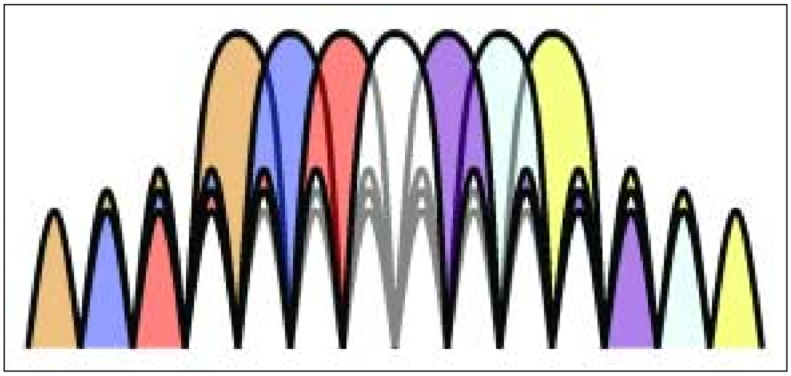
Effects of physical layer impairments on OFDM systems

The effects of common signal impairments using single-carrier modulation formats are generally well understood by system designers. The effects of these same impairments on an OFDM signal, however, can be quite different.

By Bob Cutler

The simplest way to describe an orthogonal frequency-division multiplexing (OFDM) signal is as a set of closely spaced frequency-division multiplexed (FDM) carriers. While this is a good starting point for those unfamiliar with the technology, it falls short as a model for analyzing the effects of signal impairments.

The reason it falls short is that the carriers are more than closely spaced; they are heavily overlapped. In a perfect OFDM signal, the orthogonality property prevents interference between overlapping carriers. This is different from the FDM systems we're all familiar with. In FDM systems, any overlap in the spectrums of adjacent signals will result in interference. In OFDM systems, the carriers will interfere with each other only if there is a loss of orthogonality. So long as orthogonality can be maintained, the carriers can be heavily overlapped, allowing increased spectral efficiency.



A typical set of OFDM carriers.

This article will address modulator and demodulator impairments. It will discuss how these impairments affect OFDM systems and, where appropriate, how this is different from the effect on single-carrier modulation formats.

Many theoretical papers already exist that discuss impairments and their impact on bit error rate (BER). This article will instead attempt to discuss how an impairment introduces error into a signal — something that could be infinitely more useful when performing real-world RF design and troubleshooting.

Table 1 lists a variety of common analog signal impairments and their effects on both OFDM signals and the more familiar single-carrier modulations such as quadrature phase-shift keying (QPSK) or 64-QAM (quadrature amplitude modulation). Most of these impairments can occur in either the transmitter or the receiver.

IQ imperfections

For cost reasons, analog in-phase and quadrature (I/Q) modulators and demodulators are often used in transceivers — especially for wide bandwidth signals. Being analog, these I/Q modulators and demodulators usually have imperfections that result in an imperfect match between the two baseband analog signals, I and Q, which represent the complex carrier. For example, gain mismatch might cause the I signal to be slightly smaller than the Q. In a single-carrier modulation system, this results in a visible distortion in the constellation — the square constellation of a 64-QAM signal would become rectangular.

To better understand how gain imbalance will affect an OFDM signal, look at the equations describing each individual subcarrier. In the following analysis, it's important to keep in mind that, while we are analyzing individual subcarriers, the IQ gain imbalance error is on the signal that is the composite of all subcarriers.

In Equation 1, $C_{k,m}$ is a complex number representing the location of the symbol within the constellation for the k^{th} subcarrier at the m^{th} symbol time. For example, if subcarrier k is binary-phase-shift-keying (BPSK) modulated, then $C_{k,m}$ might take on values of $\pm 1 + j0$. The complex exponential portion of Equation 1 represents the k^{th} subcarrier, which is amplitude- and phase-modulated by the symbol $C_{k,m}$. Therefore:

$$C_{k,m}(e^{j2\pi k\Delta ft}) \qquad (1)$$

Using Euler's relation, Equation 1 can be rewritten as:

$$C_{k,m}(\cos(2\pi k\Delta ft) + j\sin(2\pi k\Delta ft)) \qquad (2)$$

Now add the term "β" to represent gain imbalance. For a perfect signal, set $\beta = 0$. As shown, the gain imbalance term will also produce a gain change. This was done to simplify the analysis. Therefore:

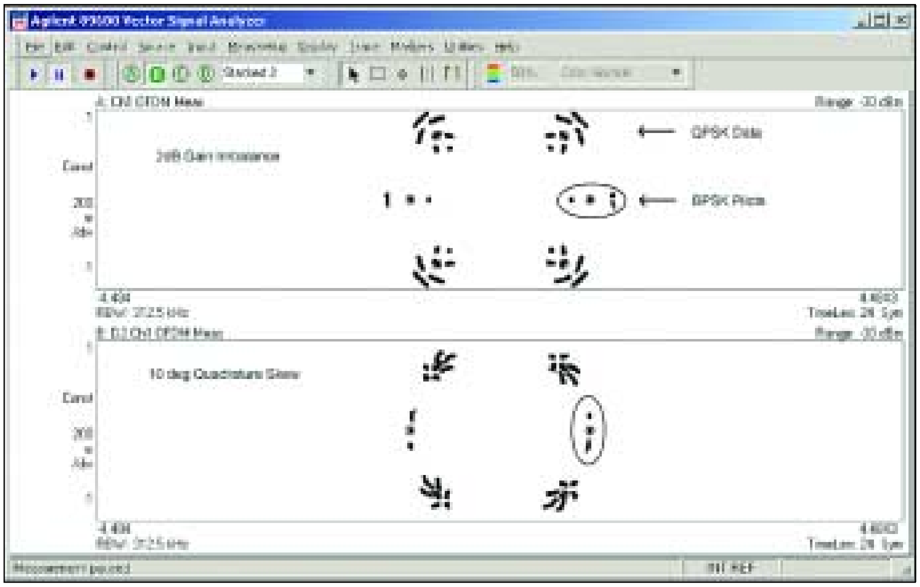


Figure 2. IQ impairments cause pilot and data spreading for OFDM signals.

$$C_{k,m}\begin{pmatrix}(1+\beta)\cos(2\pi k\Delta f t)\\ +j\sin(2\pi k\Delta f t)\end{pmatrix} \tag{3}$$

The equation can be rearranged and this can be rewritten as the sum of a perfect signal and an error signal:

$$\begin{aligned}&C_{k,m}(\cos(2\pi k\Delta f t)+j\sin(2\pi k\Delta f t))\\&+C_{k,m}\beta\cos(2\pi k\Delta f t)\end{aligned} \tag{4}$$

Finally, converting back into complex exponential notation, we get:

$$\begin{aligned}&C_{k,m}e^{j2\pi k\Delta f t}+\\&\left(C_{k,m}\frac{\beta}{2}\right)\bullet(e^{j2\pi k\Delta f t}+e^{-(j2\pi k\Delta f t)})\end{aligned} \tag{5}$$

In words, Equation 5 shows that a gain imbalance produces two error terms. The first error term is at the frequency of the k^{th} subcarrier. The second error term is at the frequency of the $-k^{th}$ subcarrier. The phase and magnitude of the error terms are proportional to the symbol being transmitted on the k^{th} subcarrier. Another way of saying this is that I/Q gain imbalance will result in each subcarrier being interfered with by its frequency mirror-image subcarrier. If you're familiar with sideband modulation, you'll instantly recognize this as imperfect sideband cancelation.

Equation 5 has several implications. First, it is generally true that for subcarriers used to carry data (as opposed to pilots), the symbol being transmitted at any given time on the k^{th} subcarrier is uncorrelated to the symbol on the $-k^{th}$ subcarrier.

For a given subcarrier, the lack of correlation from the mirror-image subcarrier implies a certain randomness to the error. This results in a spreading of the subcarrier's constellation states in a noise-like fashion. This is especially true for higher-order modulations such as 64-QAM. For lower-order modulations, such as BPSK, the error term from the mirror-image carrier has fewer states.

This can result in constellations as shown in Figure 2, where the BPSK pilot carriers of an 802.11a signal exhibit spreading that does not appear noise-like. Also, as the BPSK pilots do not have an imaginary component; the error terms associated with the pilot subcarriers are real — so the spreading is only along the real (I) axis. Note that the phase relationships between the pilot carriers in an 802.11a system are highly correlated, so the errors introduced by quadrature errors are not random.

Quadrature skew produces error terms similar to those produced by gain imbalance. Quadrature skew occurs when the two oscillators used in an I/Q modulator or demodulator do not differ by exactly 90°. For a small angular error, ϕ, it can be shown that the resulting error is orthogonal to the data. This is indicated by the j in front of the error terms in Equation 6. As with gain imbalance, the error generates energy at the k^{th} and $-k^{th}$ subcarriers. Again, the 802.11a BPSK pilots do not have an imaginary component, so the error term, which is now orthogonal, causes spreading along the Q axis as shown in Figure 2, lower trace. For the QPSK carriers in this example, the error is also orthogonal. However, unlike BPSK, a QPSK constellation doesn't look any different when rotated by 90°. (See Equation 6.):

$$\begin{aligned}&C_{k,m}e^{j2\pi k\Delta f t}+\\&j\frac{C_{k,m}\phi}{2}\bullet(e^{j2\pi k\Delta f t}+e^{j2\pi k\Delta f t})\end{aligned} \tag{6}$$

In both 802.11a and Hiperlan2, a channel estimation sequence is transmitted at the beginning of a burst. This special sequence is used to train the receiver's equalizer. The intended function of the equalizer is to compensate the received signal for multipath distortion — a linear impairment in the signal that is the result of multiple signal paths between the transmitter and the receiver. As the ideal channel estimation sequence is known by the receiver, the receiver can observe the effects of the channel on the transmitted signal and compute a set of equalizer coefficients.

In the transmitter, the channel estimation sequence is created by BPSK modulating all 52 carriers for a portion of the preamble. Not coincidentally, the equalizer consists of 52 complex coefficients — one for each subcarrier. It should come as no surprise that each subcarrier in the channel estimation sequence has the greatest influence on the equalizer coefficient computed for that same subcarrier.

The channel estimation sequence, and the receiver algorithms that compute the equalizer coefficients, are not immune from signal impairments. Consider, for example, the effect of I/Q gain imbalance on subcarriers +26 and –26 of the channel estimation sequence. Recall from Equation 5 that each subcarrier has two error terms: one at the same frequency as the subcarrier, and one at the mirror image frequency. The I/Q gain imbalance will cause mutual interference between subcarriers +26 and –26.

From the IEEE 802.11a standard, the subcarrier modulation for the channel estimation sequence is defined to be $C_{-26} = 1+j0$ and $C_{+26} = 1+j0$. Using these

Impairment	OFDM	OFDM
IQ gain balance	State spreading (uniform/carrier)	Distortion of constellation
IQ quadrature skew	State spreading (uniform/carrier)	Distortion of constellation
IQ channel mismatch	State spreading (nonuniform/carrier)	State spreading
Uncompensated frequency error	State spreading	Spinning constellation
Phase noise	State spreading (uniform/carrier)	Constellation phase arcing
Nonlinear distortion	State spreading	State Spreading (may be more pronounced on outer states)
Linear distortion	Usually no effect (equalized)	State spreading if not equalized
Carrier leakage	Offset constellation for center carrier only (if used)	Offset constellation
Frequency error	State spreading	Constellation phase arcing
Amplifier droop	Radial constellation distortion	Radial constellation distortion
Spurious	State spreading or shifting of affected subcarrier	State Spreading, generally circular

Table 1. A variety of common analog signal impairments and their effect on both OFDM signals and single-carrier modulations.

values in Equation 5, one can easily determine that the two subcarriers, when combined with the resulting error terms, will suffer an increase in amplitude. The equalizer algorithm will be unable to differentiate the error from the actual channel response, and will interpret this as a channel with too much gain at these two subcarrier frequencies. The equalizer will incorrectly attempt to compensate by reducing the gain on these subcarriers for subsequent data symbols.

The result will be different for other subcarrier pairs, depending on the BPSK channel estimation symbols assigned to each.

With QPSK subcarriers, the equalizer error caused by gain imbalance, or quadrature skew, results in seven groupings in each corner (see Figure 2). Each QPSK subcarrier suffers from QPSK interference from its mirror image. This results in a spreading to four constellation points in each corner. Each QPSK subcarrier also suffers from a bi-level gain error introduced by the equalizer. This would produce eight groupings, except that the gain error is such that corners of the groupings overlap at the ideal corner state. Only seven groupings are visible. Keep in mind that the constellation plot is an overlay of the constellations for all subcarriers. When the constellation display is limited to a single subcarrier, four groupings would be visible in each corner — with the center of the grouping displaced from the ideal corner state location.

IQ channel mismatch

When the frequency response of the baseband I and Q channel signal paths are different, an I/Q channel mismatch exists. I/Q channel mismatch can be modeled as a subcarrier-dependent gain imbalance and quadrature skew. I/Q gain imbalance and quadrature skew, as described above, are simply a degenerate form of I/Q channel mismatch in which the mismatch is constant over all subcarriers. Think of channel mismatch as gain imbalance and quadrature skew as a function of subcarrier. It is still generally true that channel mismatch causes interaction between the k^{th} and $-k^{th}$ subcarriers, but that the magnitude of the impairment could differ between the k^{th} and the $(k+n)^{th}$ carriers.

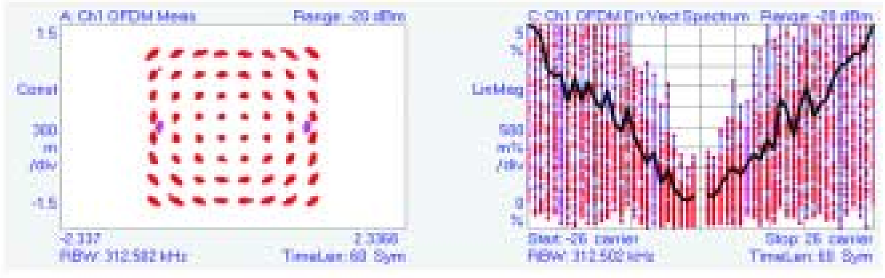


Figure 3. Error introduced by timing offsets between IQ signal paths.

Delay mismatch has a distinct error. It can occur when the signal path for the I signal differs in electrical length from the Q signal. This can be caused by different cable lengths (or traces), timing skew between D/A converters used to generate the I and Q signals, or group delay differences in filters in the I and Q signal paths.

What makes this error distinctive is that the error is greater for the outer carriers than it is for the inner carriers. In other words, the error increases with distance from the center subcarrier. In Figure 3, the constellation shows what appears to be a signal affected by phase noise. It's not.

As each symbol represents a point in both time and frequency (subcarrier), one must consider that the phase arcing may occur as a function of frequency. The plot on the right confirms that it is. This plot is the error vector spectrum. The error vector spectrum display plots signal error as a function of subcarrier number (frequency). The plot shows both the individual errors for every symbol and the RMS as a function of carrier. From this plot, it's obvious that the error increases almost linearly as a function of the distance from the center subcarrier.

In this particular example, the error results in symbols on the outer carriers having more phase error than symbols transmitted on the inner carriers. (Improper carrier spacing can have a similar effect.)

Phase noise

Phase noise results in each subcarrier interfering with several other subcarriers — especially those in close proximity. Unlike single-carrier modulations, phase noise in OFDM signals rarely results in constellation displays such as that shown in Figure 3.

There are two reasons for this. First, close-in phase noise that results in the constellation rotation for the data carriers also results in rotation of the pilot carriers. In fact, carrier phase error rotates all subcarriers by the same amount, regardless of the subcarrier frequency. Phase-tracking algorithms use the pilot symbols to detect this common rotation and compensate all of the carriers accordingly. This error is often referred to as common pilot, or common phase error (CPE). Phase noise that is not considered to be close-in results in inter-carrier interference. Instead of constellations with visible rotation, phase noise in an OFDM signal generally results in fuzzy constellation displays, similar to what would be expected if noise is added to the signal.

In 802.11a, the symbol rate is 250 kHz. As pilot symbols are present in every OFDM symbol interval, one might think of this as the sample rate at which the phase-tracking algorithms are operating. Sampling theory would suggest that one will have problems with phase noise beyond one-half the sample rate, or 125 kHz (Nyquist).

Unfortunately, there's another error mechanism that occurs at frequencies much lower than one-half the sample rate. The pilot tracking algorithms are post-fast Fourier transform (FFT). As the FFT is a mapping of the time waveform into the frequency domain (Fourier coefficients), any error that results in energy from one subcarrier leaking into other subcarriers cannot be compensated for with a simple post-FFT subcarrier de-rotation.

Phase noise modulates each of the subcarriers to the point that they no longer look like simple sinusoids within the FFT interval. Consider an FFT of a single tone. If the tone is pure and has a frequency that produces an integral number of cycles within the FFT time buffer, then an FFT will produce a result with only one non-zero value. If the pure tone is now phase modulated, it can no longer be represented as a single Fourier coefficient — even if the signal is periodic with the FFT time buffer. In other words, the subcarrier will interfere with the other subcarriers. If the phase-modulating signal is separated into a DC component plus an AC component, the DC component is what is corrected by the post-FFT pilot-tracking algorithms.

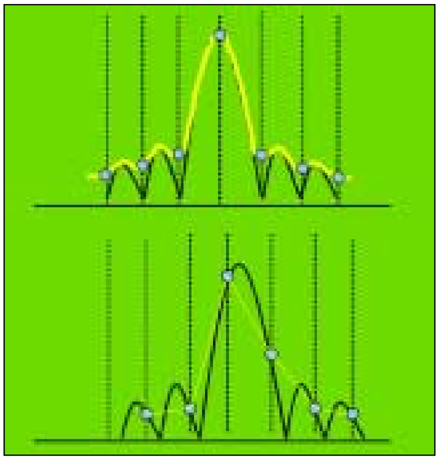


Figure 4: Intercarrier interference due to phase noise (top) and frequency error.

The impact of random phase noise on the sin(*x*)/*x* shape of an individual subcarrier is shown in Figure 4. The sin(*x*)/*x* rolloff, an artifact mostly caused by the uniform-windowed FFT used in the receiver, is gradual. Phase noise causes the nulls of the sin(*x*)/*x* spectrum to fill in, creating interference between every subcarrier and its neighbors.

Frequency error

In any coherent modulation format, it's critical that the receiver accurately tracks the transmitter frequency. Frequency is defined to be the derivative of the phase with respect to time, so frequency error can be described as a cumulative phase error that linearly increases or decreases with time, depending on the sign of the frequency error. For single-carrier modulation formats such as 64-QAM, a frequency error can be visualized as a spinning constellation diagram.

The effect of frequency error on OFDM signals is different. Under ideal conditions, each of the subcarriers in an OFDM signal is periodic within the FFT time buffer. This is critical if the subcarriers are to remain orthogonal and avoid mutual interference.

A frequency error between the transmitter and the receiver will cause all of the subcarriers to have a non-integral number of cycles in the FFT time interval, causing leakage. Figure 4 offers a graphical frequency-domain depiction of the error. A frequency error shifts the sin(*x*)/*x* spectrum of each subcarrier relative to the FFT frequency bins to the point that the spectral nulls are no longer aligned with the FFT bins. The result is that frequency error causes every subcarrier to interfere with its neighbors.

Nonlinear distortion

The effects of nonlinear distortion on an OFDM signal are easily understood by those familiar with intermodulation distortion. Nonlinear distortion is a particularly important topic for OFDM signals because the signal represents the linear summation of a large number of statistically independent subcarriers. This results in a signal with Gaussian voltage statistics on the I and Q waveforms.

Sometimes people refer to the signal as having Gaussian statistics in the context of peak-to-average power. This can be misleading because the power statistics are not Gaussian. The I and Q voltage waveforms are Gaussian, but the power, which is the sum of the squares of the I and Q signals, has a chi-square distribution. All that aside, a perfect OFDM signal can have peak envelope power that exceeds the average envelope power by more than 10 dB. This presents the power amplifier designer with some real challenges.

In the context of nonlinear distortion, it's safe to model the OFDM signal as a multitone signal with each of the tones having a random phase component. In fact, that's exactly what an OFDM signal is. Multitone analysis and testing is probably better suited to OFDM signals than to the multicarrier signals for which it was originally developed.

For most single-carrier modulation formats (excluding code-division multiple access (CDMA) signals), the Nyquist filters create peak power excursions that occur between symbols.

Errors due to saturation will cause adjacent channel power problems, but may not have a large impact on data transmission — depending on the amount of dispersion in the transmission channel and the receiver's matched filter. For OFDM, the peak power excursions may occur at any time within the symbol interval. Because the OFDM time waveform is a summation of all subcarriers, nonlinear distortion will create inter-carrier interference. Of course, it will also result in increased adjacent channel power.

Spurious signals

When a spurious signal is added to an OFDM signal, the effect is similar to what happens with single-carrier modulations. If the spurious tone is at the exact frequency of a subcarrier, then the effect is to shift the constellation of that subcarrier — just as carrier leakage shifts the constellation of a single-carrier modulation. Otherwise, the spurious signal causes a spreading of the constellation points for subcarriers near the spurious tone. Carrier leakage in an OFDM signal only affects the center carrier (if used).

Summary

OFDM signals are generated and received using many of the same components used with single-carrier modulation formats. Understanding how signal path impairments affect the OFDM signal is critical in the design and troubleshooting of OFDM systems, subsystems and components. If you are experienced with single-modulation formats, but new to OFDM, you'll want to avoid jumping to conclusions about the effect or source of signal impairments. While the source of the impairments is often the same as that for single-carrier systems, the observed effects can be quite different.

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