Crest factor analysis for complex signal processing

Understanding the complexities of summing digital signals.

By Brad Andersen

As the 21st century gains momentum, more and more of today's communications systems are using designs based on digital modulation techniques. This has increased the practice of summing several baseband signals for systems such as CDMA. Such techniques result in composite signals with large crest factors (the ratio of peak amplitude to the rms level of the signal), which can significantly affect the signal quality.

The filtering processes used in the signal generation also result in an increase of the resultant signals' crest factor. Signal fidelity is critical in many such systems, necessitating careful attention to maximizing dynamic range. In particular, optimally employing the dynamic range of digital-to-analog and back converters (DACs and ADCs) is crucial. Also important is the number of bits used for the digital signal processing functions.

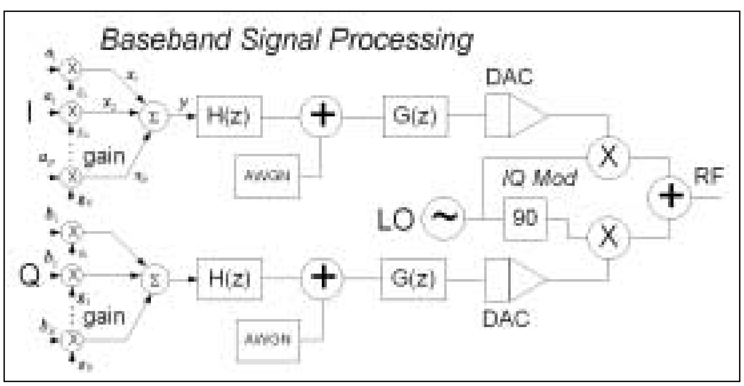

Figure 1. Baseband signal process block diagram.

The road to optimal design criterion

The designer would like to operate at the highest possible operating level to maximize the signal-to-quantization noise-level ratio (SNR). To prevent spectral splatter, the designer must not allow the signal to saturate, or worse, rollover. In the RF domain, this type of problem will cause distortion of the modulated waveform in the form of higher levels in the frequencies adjacent to the signal's occupied bandwidth and/or other spurious signals.

Tools for better design

In this article, methods will be developed for analyzing complex digital signals to establish the worst-case crest factors. These methods consider the effects of summing signals of different crest factors and different gains, as well as the effects of filtering, including interpolation filtering. With these tools, the designer can determine the precise back-off required to prevent saturation, while maximizing the SNR. For example, a spreadsheet program could be used to determine the signal's crest factor and SNR at all points within a design.

These methods will also show the designer how to determine how many bits will be required to achieve a specified SNR, and reveal interesting results such as that the worst-case crest factor has a root-sum-square behavior. Results will show that, due to digital filtering, the crest factor expansion is similar to the inverse of the noise bandwidth of the filter. The resulting RF envelope crest factor is also developed.

The analysis presented is based on the IS-2000 (cdma2000 or C2K) standard. An instrument was developed and is used to emulates a working base station for testing IS-2000-compatible mobile phones. It provides the ability to provide IS-2000 signals, plus the ability to add an additive white Gaussian noise (AWGN) signal to the composite signal. The analysis techniques were used to guarantee proper sizing of all data paths used in the CDMA baseband generation circuitry. This system is designed to provide the IS-2000 forward-modulation signals to test IS-2000 mobile phones with a number of testing scenarios.

A typical baseband signal processing scheme for IQ modulation of an RF carrier is shown in Figure 1. Each in-phase and quadrature (I and Q) signal is a summation of independent signals with individual scalings. These are summed together and filtered. The filtering may include interpolation to increase the effective sample rate of the signal, simplifying the analog reconstruction filter. Other signals can be summed followed by another stage of interpolation filtering. An AWGN signal is shown as the additional signal for the case of the test set mentioned above. These resulting signals are then used as the I and Q inputs of an IQ modulator to modulate a carrier or intermediate frequency signal.

Independence of signals is mentioned several times, but in actuality, uncorrelated signals generally satisfy the requirements for the resulting conclusions.

To examine the resulting crest factor for the signals of interest throughout the system, methods for determining the crest factor after various signal processing functions are developed. The basic assumptions are given for the signals in the analy-

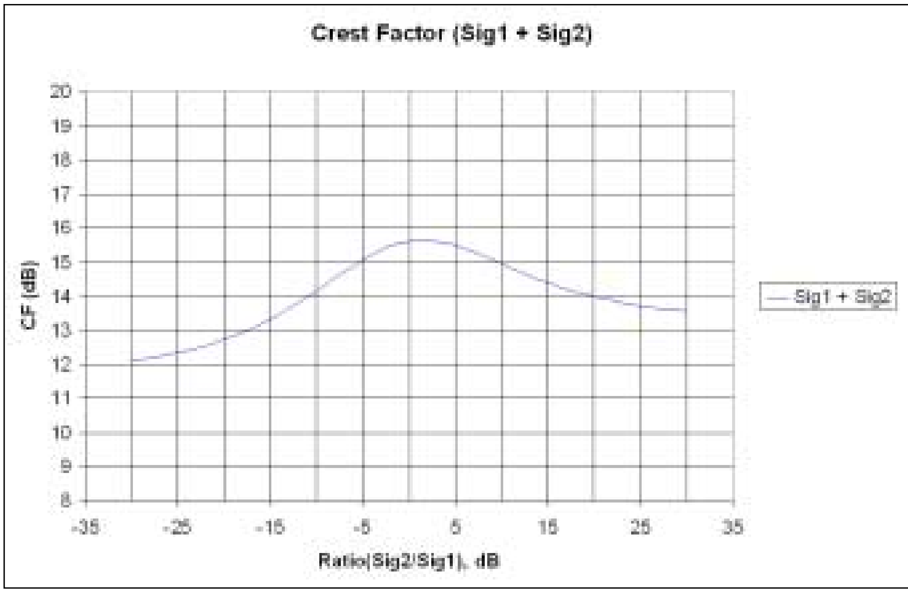

Figure 2. The crest factor of a sum of two signals as a function of their relative levels.

sis. In the given example, the analysis technique and the considerations will be addressed as well.

Variables and signal analysis

• *Baseband signal:*

Given: x_i are independent, discrete random variables with zero mean. Crest factor for a bounded, zero mean discrete random variable x can be defined as:

$$C = \frac{\max(|x|)}{\sigma_x}, \text{ or in dB,}$$

$$20\log_{10}(C)dB \text{ or } 10\log_{10}(C^2)dB$$

where :

$$\sigma_x^2 = \sum_i x_i^2 p_x(x_i),$$

and x_i are the discrete values x may exhibit and $P_x(x_i)$ is the probability of x_i occurring.

• *Crest factor of a sum of independent random variables*:

The signals in the I channel are used to develop the equations to determine the crest factor:

$$y = \sum_{i=1}^{n} x_i$$

For independent signals, it is well-known that the power of the resulting signal is the sum of the individual signal's rms power. Thus:

$$\sigma_y^2 = \sum_{i=1}^{N} \sigma_{x_i}^2$$

and the peak level is the worst case situation of adding the peak of each

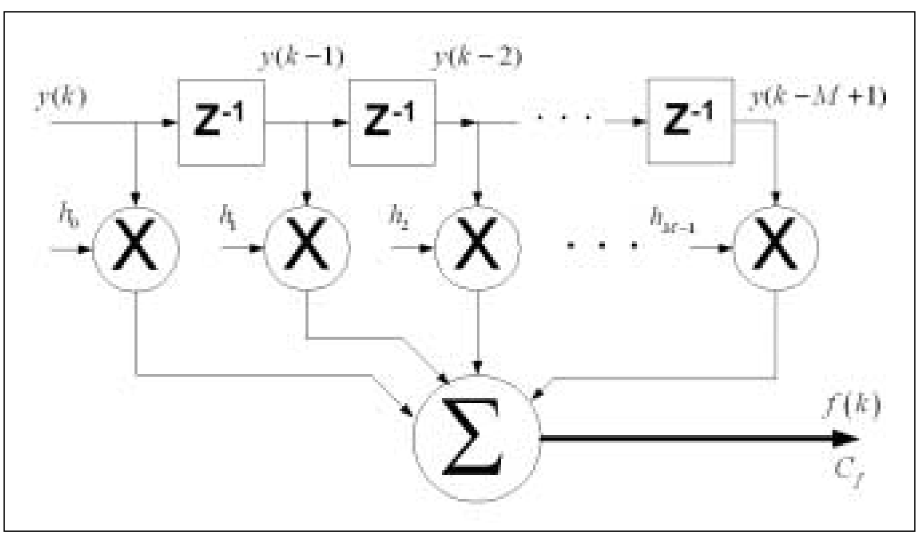

Figure 3. The baseband filter model.

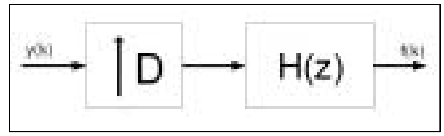

Figure 4. Interpolation/polyphase filter conversion.

input signal is:

$$\max(y^2) = \left(\sum_{i=1}^{N} \max(|x_i|)\right)^2$$

Thus, the crest factor is:

$$C_y^2 = \frac{\left(\sum_{i=1}^{N} \max(|x_i|)\right)^2}{\sum_{i=1}^{N} \sigma_{x_i}^2}$$

Summing distributed signals

Consider adding gain scaling to the signals being summed, again referring to Figure 1, with the a_i independent identically distributed (i.i.d.). Let the crest factor of a_i be C_a. And, a_i is the signal i, an independent and zero mean, and g_i is the scaling factor for the i^{th} signal.

The xi have the identical crest factor as the ai, but each xi may have a different variance, σx_i^2.

Next, look at determining the crest factor for the sum:

$$y = \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} g_i a_i$$

Because the a_i are identically distributed, the previous results may be applied:

$$\max(|y|) = \sum_{i=1}^{N} \max(|x_i|) =$$

$$\sum_{i=1}^{N} |g_i| \max|a_i| = \max|a| \sum_{i=1}^{N} |g_i|$$

The variance of y is given from before as:

$$\sigma_y^2 = \mathrm{E}\{Y^2\} = \mathrm{E}\left\{\left(\sum_{i=1}^{N}(x_i)\right)\right\} = \sum_{i=1}^{N} \mathrm{E}\{x_i^2\}$$

$$= \sum_{i=1}^{N} \mathrm{E}\{g_i^2 a_i^2\} = \sum_{i=1}^{N} g_i^2 \mathrm{E}\{a_i^2\} = \sum_{i=1}^{N} g_i^2 \sigma_2^a$$

$$= a_a^2 \sum_{i=1}^{N} g_i^2$$

Therefore, C_y^2 is given by:

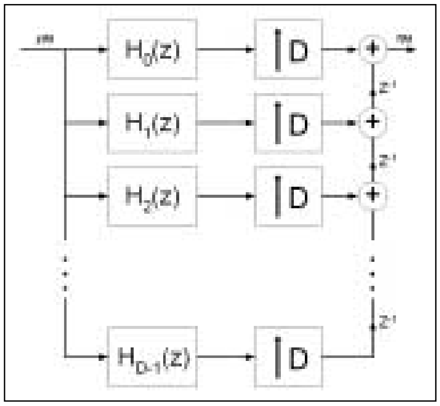

Figure 5. Polyphase filter representation.

$$C_y^2 = \frac{\max\left(|a|^2\right)\left(\sum_{i=1}^{N}|g_i|\right)^2}{\sigma_a^2\sum_{i=1}^{N} g_i^2} = C_a^2\frac{\left(\sum_{i=1}^{N}|g_i|\right)^2}{\sum_{i=1}^{N}|g_i^2|}$$

The crest factor of the sum is multiplied by an expansion factor that is a simple function of the gains. It is always greater than or equal to one.

Worst-case signal combinations

It is interesting (and handy) to determine what set of gains produces the worst-case crest factor. First, consider a sum using two signals with different crest factors. What are the gain settings that will maximize the resulting signal's crest factor? By using calculus to find maximum value of a function, the condition for maximum crest factor can be found using the following summing equation:

$$C_A^2 = \frac{(C_1\sigma_1 + C_2\sigma_2)^2}{\sigma_1^2 + \sigma_2^2}, \text{ since}$$

$$C_1 = \frac{P_1}{\sigma_1} \text{ and } P_1 = C_1\sigma_1$$

and rearranging terms:

$$C_A^2 = \frac{C_1^2 + C_2^2\dfrac{\sigma_2^2}{\sigma_1^2} + 2C_1C_2\sqrt{\dfrac{\sigma_2^2}{\sigma_1^2}}}{1 + \dfrac{\sigma_2^2}{\sigma_1^2}}$$

This analysis shows that the maximum crest factor for the sum of two signals occurs when:

$$\frac{\sigma_1}{\sigma_2} = \frac{C_1}{C_2} \text{ or } \frac{\sigma_1}{C_1} = \frac{\sigma_2}{C_2}$$

When this condition is satisfied, the resulting crest factor is found to be:

$$C_A^2 = C_1^2 + C_2^2$$

(Note the similarity to the root sum squares.)

Figure 2 shows the crest factor of a sum of two signals as a function of their relative levels. The peak value occurs when the ratio of the rms levels equals the ratio of the crest factors, as discussed above. It may also be observed that when the summed signal is made up primarily of one of the input signals, the crest factor approaches that signal's crest factor, as would be expected.

The crest factors used in the plot are two signals summed together: signal 1, CF = 11.8 dB; and signal 2, CF = 13.33 dB. The peak crest factor is 15.64 dB when the ratio is 1.53 dB (13.33 - 11.8 dB).

By using mathematical induction, the condition for maximum crest factor when summing N signals can be found. The maximum crest factor for the sum of N signals with different crest factors occurs when the signals satisfy:

$$\frac{\sigma_i}{C_i} = \frac{\sigma_j}{C_j},$$

for any i and j. Hence:

$$C_{sum}^2 \le \sum_{i=1}^{N} C_i^2,$$

achieves equality when the signals satisfy the specified ratios. And the maximum crest factor for the sum of N signals, each with the same crest factor, occurs when all the gains are equal. In this case, the crest factor expansion is given by:

$$C_y^2 = C_a^2\frac{g^2\left(\sum_{i=1}^{N}1\right)^2}{g^2\sum_{i=1}^{N}1} = C_a^2\frac{N^2}{N} = C_a^2 N$$

Hence, the maximum crest factor for the sum of N i.i.d. signals, each with the same crest factor, is $\sqrt{N}$, or ($10\log_{10} N$) greater than the crest factor of an individual signal.

Baseband filtering

Given an input signal stream $y(n)$ where n is the time variable with the following characteristics:

* Zero mean.
* $y(n)$ i.i.d. $y(m)$, $n \neq m$.
* Each $y(n)$ has crest factor C_y.

Figure 3 represents the baseband filter model. Due to the independence of the successive $y(k)$, the inputs to the filter's summer satisfy the conditions previously used. Observe by inspection that:

$$C_f^2 = C_y^2\frac{\left(\sum_{i=0}^{M-1}h_i\right)^2}{\sum_{i=0}^{M-1}h_i^2}$$

Note that this expansion factor is closely related to the reciprocal of the equation for the noise bandwidth, B_N, of a low-pass digital filter:

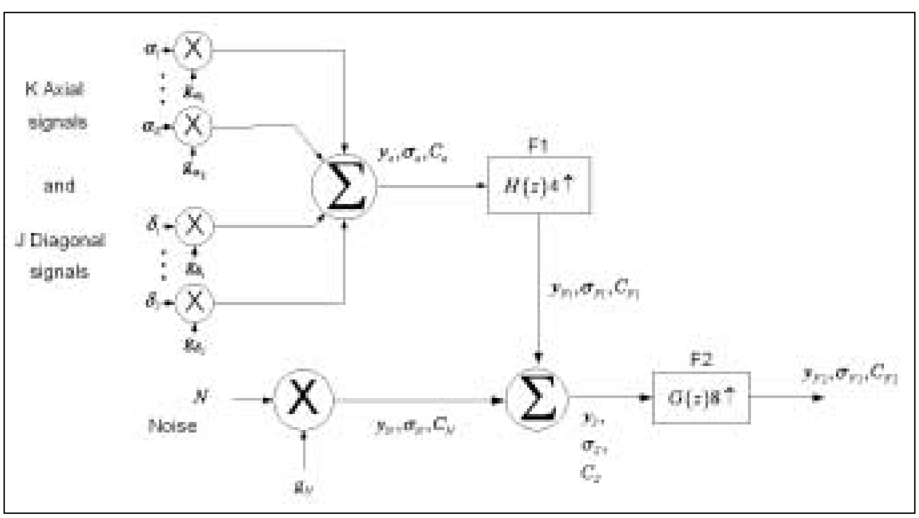

Figure 6. One-half of the example baseband modulate signal.

All Diagonal, CF = 1	Worst Case Levels	Typical IS-95 Setup
Channel	**Level (dB)**	**Level (dB)**
Pilot	-10	-7
Sync	-10	-16
Paging	-10	-12
Traffic1	-10	-15.6
Traffic2	-10	-15.6
Traffic3	-10	-15.6
Traffic4	-10	-15.6
Traffic5	-10	-15.6
Traffic6	-10	-15.6
OCNS	-10	-2.62
CF of different signals & components	**CF (dB)**	
Diagonal signals	0	
Noise signal, y_N	10	
Noise signal after F2, y_{NF1}	13.71	
Filter F1 Expansion Factor	6.97	
Filter F2 Expansion Factor	3.71	
Convolved Filter F12 Expansion Factor	7.22	
Location of determined CF	**CF(dB)**	**CF(dB)**
Sum of signals, y_a	10	8.27
Output of Filter F1, y_{F1}	16.97	15.25
Output of Filter F2, y_{F2}, w/o noise	17.22	15.5
Sum of signal & noise, y_T with $\frac{\sigma_N}{\sigma_{F1}}$ as shown	17.76 $\frac{\sigma_N}{\sigma_{F1}} = -6.97\,dB$	15.86 $\frac{\sigma_N}{\sigma_{F1}} = +1dB$
Output of Filter F2, y_{F2}, w/ noise	18.82 $\frac{\sigma_{NF1}}{\sigma_{sF2}} = -3.51\,dB$	17.6 $\frac{\sigma_{NF1}}{\sigma_{sF2}} = +1dB$

Traffic: Axial, CF=1.414 Others: Diagonal, CF=1	Worst Case Levels	Typical IS-2000 Setup
Channel	**Level (dB)**	**Level (dB)**
Pilot	-12.04	-7
Sync	-12.04	-16
Paging	-12.04	-12
Traffic1 (CF=1.414)	-9.03	-15.6
Traffic2 (CF=1.414)	-9.03	-15.6
Traffic3 (CF=1.414)	-9.03	-15.6
Traffic4 (CF=1.414)	-9.03	-15.6
Traffic5 (CF=1.414)	-9.03	-15.6
Traffic6 (CF=1.414)	-9.03	-15.6
OCNS	-12.04	-2.62
CF of different signals & components	**CF (dB)**	
Diagonal signals	0	
Axial signals	3	
Noise signal, y_N	10	
Noise signal after F2, y_{NF1}	13.71	
Filter F1	6.97	
Filter F2	3.71	
Convolved Filter F12	7.22	
Location of determined CF	**CF(dB)**	**CF(dB)**
Sum of signals, y_a	9.61	8.89
Output of Filter F1, y_{F1}	16.58	15.86
Output of Filter F2, y_{F2}, w/o noise	16.83	16.11
Sum of signal & noise, y_T with $\frac{\sigma_N}{\sigma_{F1}}$ as shown	17.44 $\frac{\sigma_N}{\sigma_{F1}} = -6.58\,dB$	16.25 $\frac{\sigma_N}{\sigma_{F1}} = +1dB$
Output of Filter F2, y_{F2}, w/ noise	18.55 $\frac{\sigma_{NF1}}{\sigma_{sF2}} = -3.12\,dB$	17.92 $\frac{\sigma_{NF1}}{\sigma_{sF2}} = +1dB$

Table 1. Crest factor at points within the topology.

$$B_N = \frac{\sum_i h_i^2}{\left(\sum_i h_i\right)^2}$$

However, the crest factor expansion factor uses the sum of the absolute values of the filter's coefficients, where the noise bandwidth uses the sum of the filter's coefficients. Hence, the crest factor expansion factor is always greater than the reciprocal noise bandwidth factor.

Interpolation filters

The effects on crest factor by interpolation filters may be analyzed as follows: Start with a general form for an interpolating filter and transform it into a polyphase representation (see Figure 4). Then apply the previously developed crest factor analysis to the resulting filter topology.

Given:

$$H(z) = \sum_{j=0}^{DE-1} h_j z^{-j}$$,

H(z) has (DE) coefficients (may have to zero the pad to get an integer multiple of D). This interpolation filter can be transformed into a polyphase representation as shown in Figure 5.

Each branch of the polyphase filter uses coefficients from the original filter in the following manner, e.g., the d^{th} filter:

$$H_d(z) = \sum_{i=0}^{E-1} h_{d,i} z^{-i} = \sum_{i=0}^{E-1} h_{iD+d}\, z^{-i}$$

for $d = 0$ to $D - 1$

The input sequence $y(k)$ has the same characteristics as the previous analysis. To determine the crest factor, the peak value is the maximum absolute value from among the branch filter outputs. The peak value is found by examining each path's filter: Peak_d = peak value of branch *d*. The maximum possible output value squared is then given by:

$$\max_d \left[(Peak_d)^2\right] = \max_d \left[\left(\sum_{i=0}^{E-1} |h_{d,i}|\right)^2\right] \max |y|^2$$

And, the average output power (or variance) is the average of the branch filter outputs given by:

$$\sigma_f^2 = \frac{1}{D}\left[\begin{matrix}\sum_{i=0}^{E-1} h_{0,i}^2 + \sum_{i=0}^{E-1} h_{1,i}^2 + \\ \sum_{i=0}^{E-1} h_{2,i}^2 + \ldots \sum_{i=0}^{E-1} h_{D-1,i}^2\end{matrix}\right]\sigma_y^2 = \frac{\sigma_y^2}{D}\sum_{i=0}^{DE-1} h_i^2$$

Therefore, the crest factor is:

$$C_f^2 = C_y^2 \frac{\max_d \left[(Peak_d)^2\right]}{\sigma_f^2}$$

$$= C_y^2 D \frac{\max_d \left[\left(\sum_{i=0}^{E-1} h_{d,i}\right)^2\right]}{\sum_{i=0}^{DE-1} h_i^2}$$

The denominator is the summation of the coefficients squared of the filter $H(z)$. The numerator will depend on

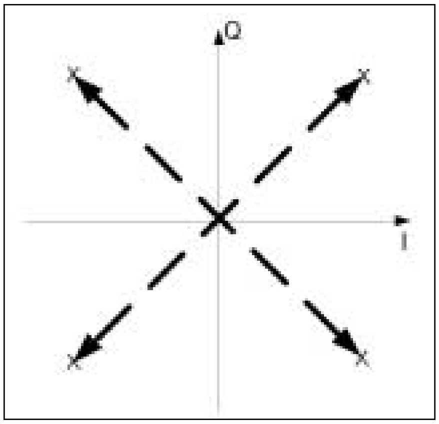

Figure 7. Special case 1: the diagonally aligned IQ constellation.

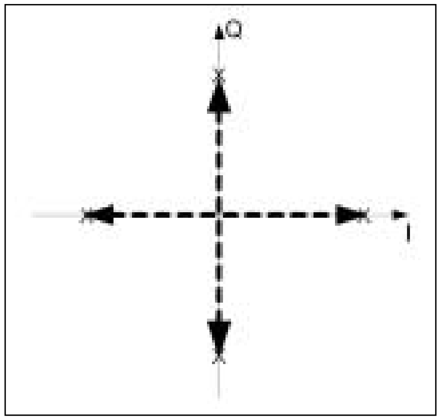

Figure 8. Special case 2: the axially aligned IQ constellation.

how the polyphase filters' coefficient magnitudes sum together.

An example system

This example is based on the IS-2000 standard. It allows the summing of two sets of inputs, diagonally and axially aligned IS-2000 signals with different crest factors. The diagonal signals are so called because the complex I and Q signals result in a constellation at diagonal points in the IQ plane. This is illustrated below as special case 1.

Special case 2 illustrates the axial signals. The axial signals result in a four-point constellation on the I and Q axes of the IQ plane. The I and Q components are not independent from one other, but each (I or Q) comprises independent streams of data.

The summed IS-2000 signals are filtered with the IS-2000 baseband interpolating filter. This output is summed with an AWGN signal with a subsequent filter that sets the noise bandwidth and further interpolates the signal. This system is used to produce the forward modulation signals for testing IS-2000 mobile phones. Figure 6 presents only one-half of the baseband modulation signal (I or Q) because each half is processed separately and identically.

Filter F1 ($H(z)4\uparrow$) (see Figure 6) is patterned after the one specified in IS-2000, which sets the bandwidth of the IS-2000 forward channel signals and interpolates the data by a factor of four. Filter F2 ($G(z)8\uparrow$) (see Figure 6) is used to set the bandwidth of the AWGN signal and interpolates the data by a factor of eight. This simplifies the reconstruction filter following the DACs that convert the digital baseband I and Q signals to analog format.

The noise signal employed is a truncated Gaussian amplitude distribution and i.i.d. with a crest factor of about 10 dB. Although this may appear low for a noise approximation, the effective crest factor of the noise is increased by the filtering applied. The mobile unit's own receiver processing further increases the effective noise crest factor. All this allows for a noise source with a relatively low initial crest factor.

I and Q alignment – special cases

The IS-2000 signals have two alignments for the I & Q signals, and they have different crest factors.

• *Special case 1:* the diagonally aligned IQ constellation (see Figure 7).

From this, it can be seen that the I (or Q) signal has the following characteristics:

$$\delta = 8 = \begin{cases} +1 \\ -1 \end{cases}$$

and equally probable, thus max

$$(|\delta_i|) = 1, \sigma_\delta^2 = 1, \sigma_\delta = 1 \Rightarrow C_\delta = 1, i = 1 \text{ to } J$$

Also, the I and Q channels are independent (uncorrelated).

• *Special case 2*: The axially aligned constellation (see Figure 8).

Here, the I or Q components have the following characteristics:

$$\alpha_i = \begin{cases} +1, pdf = 0.25 \\ 0, pdf = 0.5 \\ -1, pdf = 0.25 \end{cases}$$

Then,

$$\max(|\alpha_i|) = 1$$

$$\sigma_\alpha^2 = \sum_{i=1 \text{ to } k} \alpha_i^2 p(\alpha_i)$$

$$\sigma_\alpha^2 = \frac{1}{4}(-1)^2 + \frac{1}{2}(0)^2 + \frac{1}{4}(+1)^2 = \frac{1}{2}$$

$$C_\alpha^2 = \frac{[\max(|\alpha|)]^2}{\sigma_\alpha^2} = \frac{1}{\frac{1}{2}} = 2$$

$$C_\alpha^2 = 2 \bullet C_\delta^2$$

The I and Q constituents for axial signals have a 3 dB higher crest factor than that for diagonal signals.

Unfortunately, the I and Q channels are not mutually independent. Whenever the I channel is non-zero, the Q channel is zero and vice-versa. This is only a consideration when the crest factor of the RF envelope signal is considered later.

The crest factor of the signal as it progresses through the stages of the modulation path can be determined by examining the various signals separately and then combining them using the analysis technique previously developed. Figure 9 models the signal flow that is under examination.

The solution steps

From the definition of crest factor, $\text{peak}_x = C_x \sigma_x$, then:

$$C_\alpha^2 = \frac{\left(\sum_{i=1}^{J} |C_\delta \sigma_{\delta_i}| + \sum_{i=1}^{K} |C_{\alpha_i} \sigma_{\alpha_i}|\right)^2}{\sum_{i=1}^{J} \sigma_{\delta_i}^2 + \sum_{i=1}^{K} \sigma_{\alpha_i}^2}$$

$$= \frac{\left(C_\delta \sum_{i=1}^{J} \sigma_{\delta_i} + C_\alpha \sum_{i=1}^{K} \sigma_{\delta_i}\right)^2}{\sum_{i=1}^{J} \sigma_{\delta_i}^2 + \sum_{i=1}^{K} \sigma_{\alpha_i}^2}$$

Solve for crest factor at output of H(z)

$$C_{F1}^2 = C_a^2 D \frac{\max_d \left(\sum_{i=0}^{E-1} |h_{d,i}|\right)^2}{\sum_{i=0}^{DE-1} h_i^2}, D = 4$$

Solve for crest factor at input of G(z)

$$C_S^2 = \frac{(C_{F1}\sigma_{F1} + C_N \sigma_N)^2}{\sigma_{F1}^2 + \sigma_N^2}$$

Solve for crest factor at output of G(z)

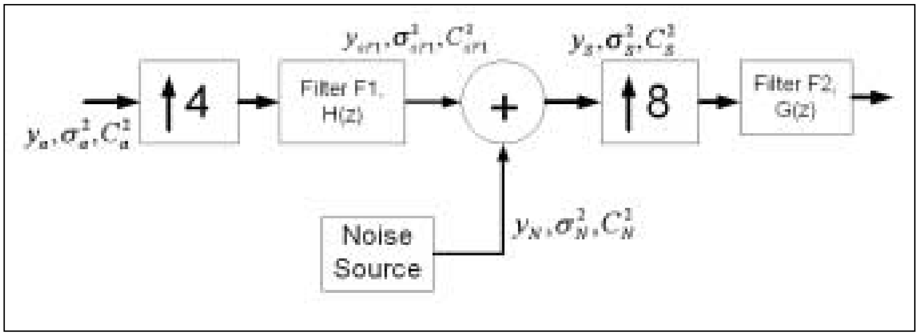

Figure 9. Crest signal modulation path flow.

$$C_{F2}^2 = \frac{(C_{aF12}\sigma_{aF12} + C_{NF2}\sigma_{NF2})^2}{\sigma_{aF12}^2 + \sigma_{NF2}^2}$$

Because the output of $H(z)$ is no longer uncorrelated data, it cannot be used as the input to $G(z)$ with the technique previously presented here. The model of the circuit must be changed to combine the two filters for the IS-2000 signals before summing with the noise after it has been filtered with $G(z)$. Using the noble identities, the following transformation can be made to shift $H(z)$ to the right of the 8x upsampler. Then $G(z)$ is convolved with $H(z^8)$. This allows computing the crest factor of the IS-2000 signals at the output of $G(z)$, (y_{aF12}). After the noise signal is passed through, G(z) is available to add to the IS-2000 signal for the final crest factor calculation (see Figure 10).

Example systems' crest factors

Table 1 illustrates what crest factors are encountered at various points in the topology for some different signal gain settings and signal types. First, the conditions for all diagonal signals are examined with the worst-case situation, then a typical setup is used in IS-95 (cdmaOne) testing. Second, the condition where some of the signals are axially aligned is examined, again with the worst-case situation. Then a typical setup is used in IS-2000 testing.

All diagonal and typical IS-95

The levels used for the worst-case situation satisfy the maximum crest factor criteria at the point the signals are summed together. This is shown by the different ratios

$$\frac{\sigma_N}{\sigma_{Fx}},$$

used in the two locations where the noise and signal are summed together, namely before and after Filter F2. The typical setup uses the same ratio, +1 dB, as is specified for several IS-98 tests.

Diagonal, axial and typical IS-2000

Note the levels used for the worst-case situation. They are adjusted to meet the max crest factor criteria; the ratio of levels equals the ratio of crest factors. Note also that the worst-case diagonally aligned signals have a larger crest factor than the worst case with six axially aligned among the 10 signals. However, the IS-2000 typical case with the axially aligned traffic signals has a slightly larger crest factor than the IS-95 situation with the diagonally aligned traffic signals.

Complex summation, translation to RF

The crest factor of the baseband signals has been examined, so it is necessary to discuss the crest factor of the modulated RF signal. The RF signal is modulated by using an IQ modulator, where the baseband signals are applied to the I & Q inputs. To more easily examine this process, an equivalent model using complex representation is used.

Referring to Figure 11:

$$S(t) = I(t) + jQ(t)$$
$$Y(t) = S(t)e^{j(\omega_c t+\theta)}$$
$$W(t) = \mathrm{Re}\left[S(t)e^{j(\omega_c t+\theta)}\right]$$

First, find the average values:

$$E\{|S(t)|^2\} = E\{S(t)S^*(t)\}$$
$$= E\{I^2(t) + Q^2(t)\} = \sigma_I^2 + \sigma_Q^2 = 2\sigma^2$$

Note that the rotation by the complex exponential does not change the mean square value:

$$E\{(W(t))^2\} = E\left\{\left(\frac{(Y)(t) + Y^*(t)}{2}\right)^2\right\}$$
$$= \frac{1}{4}E\{(Y(t))^2 + (Y^*(t))^2 + 2Y(t)Y^*(t)\} \rightarrow$$
$$E\{(Y(t))^2\} = 2E\{e^{j(\omega_c t+\theta)}\}E\{(S(t))^2\} = 0$$

Similarly,

$$E\{(Y^*(t))^2\} = 0$$

Therefore,

$$E\{(W(t))^2\} = \frac{1}{2}E\{Y(t)Y^*(t)\}$$
$$= \frac{1}{2}E\{|Y(t)|^2\} = \frac{2\sigma^2}{2} = \sigma^2$$

Now find the peak value:

$$\max\left[(W(t))^2\right] = \max\left[(\mathrm{Re}(Y(T)))^2\right]$$
$$= \max\left[\mathrm{Re}(S(t)e^{j(\omega_c t+\theta)})^2\right]$$

But the complex exponential has magnitude 1 and simply rotates the complex signal $S(t)$ at frequency ω_c. So the maximum instantaneous squared value of the output occurs when the maximum magnitude amplitude of the signal occurs and the complex exponential has simultaneously rotated it to be along the real axis. This yields:

$$\max\left[(W(t))^2\right] = \max\left[|S(t)|^2\right]$$
$$= \max\left[I^2(t) + Q^2(t)\right]$$

If $I(t)$ and $Q(t)$ are independent:

$$\max\left[I^2(t) + Q^2(t)\right] = \max\left[I^2(t)\right]$$
$$+ \max\left[Q^2(t)\right]$$
$$C_I^2 = \frac{\max\left[I^2(t)\right]}{\sigma^2}$$

Therefore:

$$\max\left[I^2(t)\right] = C_I^2\sigma^2$$

and:

$$\max\left[Q^2(t)\right] = C_Q^2\sigma^2$$

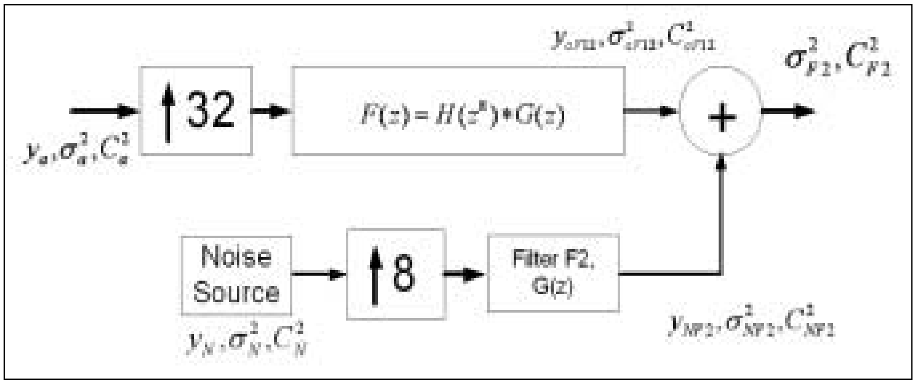

Figure 10. Crest signal modulation path flow: *G(z)* convolved with *H(z^8)*.

Peak crest factor

Next the peak crest factor for the RF signal, $W(t)$, is addressed:

$$\text{Peak } C_w^{\,2} = \frac{\max\left[(W(t))^2\right]}{E\{(W(t))^2\}}$$

$$= \frac{C_i^{\,2}\sigma^2 + C_q^{\,2}\sigma^2}{\sigma^2} = 2C_i^{\,2} \text{ (since } C_i^{\,2} = C_q^{\,2})$$

This is the ratio of the instantaneous peak voltage squared to the average output power, but for translated signals (IF or RF) it is customary to consider the crest factor for the envelope signal. Because most RF devices are characterized with sinusoidal signals at either specified or measured power levels, having the crest factor as a function of a sinusoidal signal's power level is more appropriate.

Envelope crest factor

To determine the envelope crest factor, first determine the power of a sinusoidal signal that has the same peak voltage as the modulated signal's peak voltage. This value is then divided by the average power of the modulated signal. The result is defined as the translated signal's envelope crest factor. (E*nvelope* $C_W^{\,2}$ equals *the power of the sinewave with the peak value of the modulated signal divided by the average power of the modulated signal*.), or:

$$C_w^{\,2} = \frac{\left(\frac{\max(|W|)}{\sqrt{2}}\right)^2}{\sigma_w^{\,2}}$$

$$= \frac{\frac{1}{2}\max\left((W)^2\right)}{\sigma_w^{\,2}} = \frac{1}{2}(2C_i^{\,2}) = C_i^{\,2}$$

Thus, the envelope crest factor of the RF signal is the same as the crest factor of the baseband quadrature components when the I and Q channels are uncorrelated.

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Correlated signals

What happens when the signals are correlated? Consider when I and Q are the same — perfectly correlated. Repeating the analysis yields the same result. The envelope crest factor is the same as the crest factor of I(t) (and Q(t)). In general, it is asserted that the envelope crest factor is never greater than the crest factor of the individual I and Q components. However, there are signals with an envelope crest factor less than the constituent I, Q crest factors.

A prime example of this is the axially aligned signal. Recall that the I and Q components have a crest factor of √2 (or 3 dB). However, this signal is just a 45° rotation of the diagonally aligned signal that has a crest factor of 1. By inspection, the envelope crest factor of this signal is 1, while its I and Q crest factors are √2 . Hence, the envelope crest factor is 3 dB lower than that of the I (or Q) crest factor. This occurs because, when the I channel has a non-zero value, the Q channel value is zero and vice-versa. While it has not been

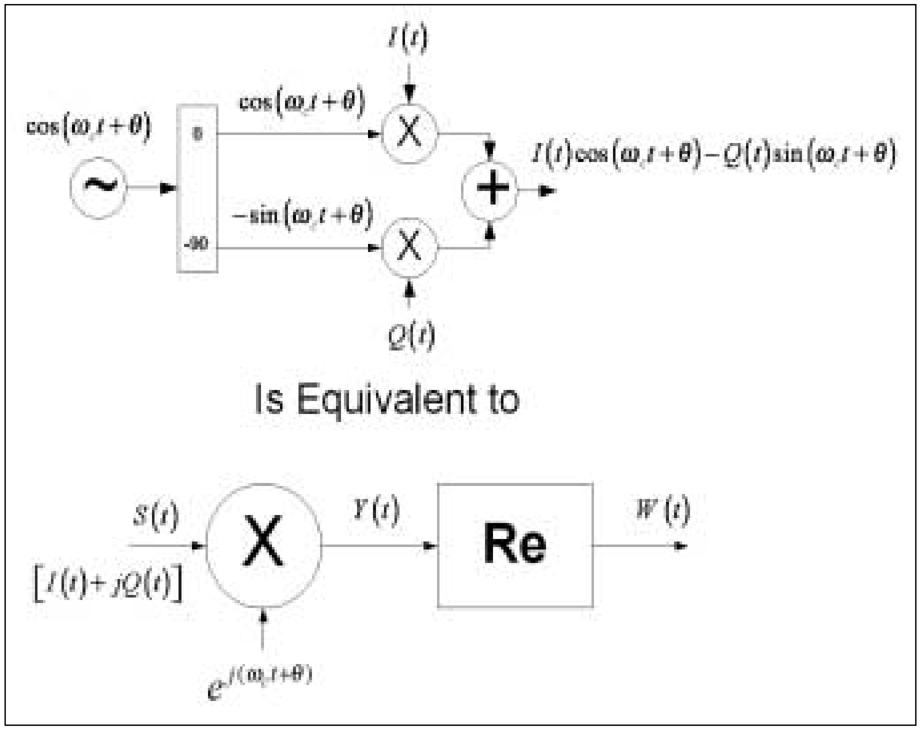

Figure 11. Complex summation and translation to RF.

proven, it is believed that these cases set the extremes, i.e., the envelope crest factor is never greater, nor 3 dB less, than the I (and Q) channel's crest factor.

Conclusion

It has been shown how to determine the worst-case crest factor for a signal formed by summing multiple independent and uncorrelated signals with arbitrary crest factors and gains. This is useful in determining the dynamic range necessary at any point in the digital signal processing data path.

The examination has included how to deal with digital filtering and interpolation along with crest factor and gain effects. The examples that were given show how the crest factor for a composite signal increases substantially when several signals are summed together over the crest factor of one of the input signals. The worst-case situation has also been identified to allow the designer knowledge of how much range is required to be assured that the signal is never limited. Finally, the translation to an IF or RF signal was discussed, showing the crest factor for the envelope signal is generally the same as the baseband signals' crest factor. There are some instances where the crest factor can be as much as 3 dB less, but this is unusual.

RF

Acknowledgments

The author is deeply indebted to George Moore for providing a great deal of guidance for and review of this paper. Also, prior to this paper, Bill Burns implemented much of this analysis in spreadsheet form during development of the Agilent Technologies 8960 Wireless Communications Test Set (E5515C).

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