Comparing windows for multitone suppression

Harmonic interference suppression requires windows with fast decaying sidelobes. When the requirement is to keep peak sidelobes low, Harris windows are preferred; but when the mainlobes are to be of minimum widths, the recently published Kulkarni windows offer a good choice.

By Raghavendra G. Kulkarni

Interfering signals of harmonic nature, if not suppressed sufficiently, degrade the performance of the receiver. To suppress such signals, it is necessary to use windows that have fast decaying sidelobes. White papers by Harris[1] and Nuttall[2] provide an exhaustive survey of window functions.

The windows that have faster decay of sidelobes (ie., 12 dB/octave or more) and that are expressed in the form of a series function are considered here. These window functions are compared in terms of mainlobe width, peak sidelobe and the asymptotic decay. The mainlobe width of a window function increases as the number of terms in the series is increased. In general, there will be trade-off between sidelobe level and the mainlobe width—and between asymptotic decay and the sidelobe level.

When the requirement is to suppress the nearby and far-off tones of broadband signals, Harris windows suit the application as they have wide mainlobes with low sidelobe peaks. The three-term Harris window has 18 dB/octave decay of sidelobes with a sidelobe peak of -64.2 dB, whereas the four-term Harris window provides a sidelobe peak of -93.3 dB for the same asymptotic decay. However, the four-term function has wider mainlobe width than that of three-term function.

If the signals encountered are narrowband, then the wide mainlobe of the Harris window allows more noise to pass through, degrading the signal-to-noise ratio (SNR). In such a situation, windows with narrower mainlobes are to be employed. Cosine powered windows offer narrower mainlobes than Harris windows—at the cost of higher sidelobe peaks.

The recently published Kulkarni windows[6,7] offer the narrowest possible mainlobe widths for a desired asymptotic decay. In fact, the mainlobe of the Kulkarni window can be made as narrow as that of a rectangular window (±1/T). However, hardware implementation becomes the limiting factor.

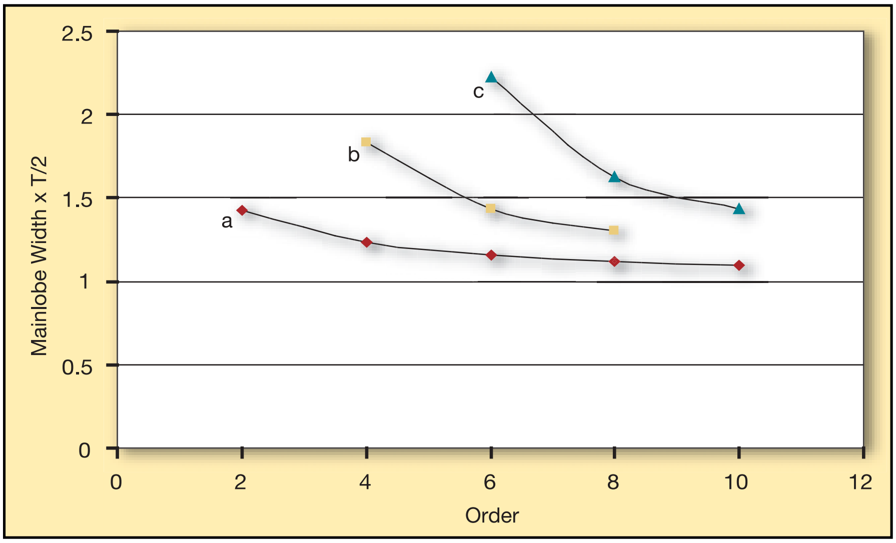

Figure 1. Plot of mainlobe width vs. order for the Kulkarni window series for asymptotic decays of: a. 12 dB/octave (two-term series), b. 18 dB/octave (three-term series), and c. 24 dB/octave (four-term series).

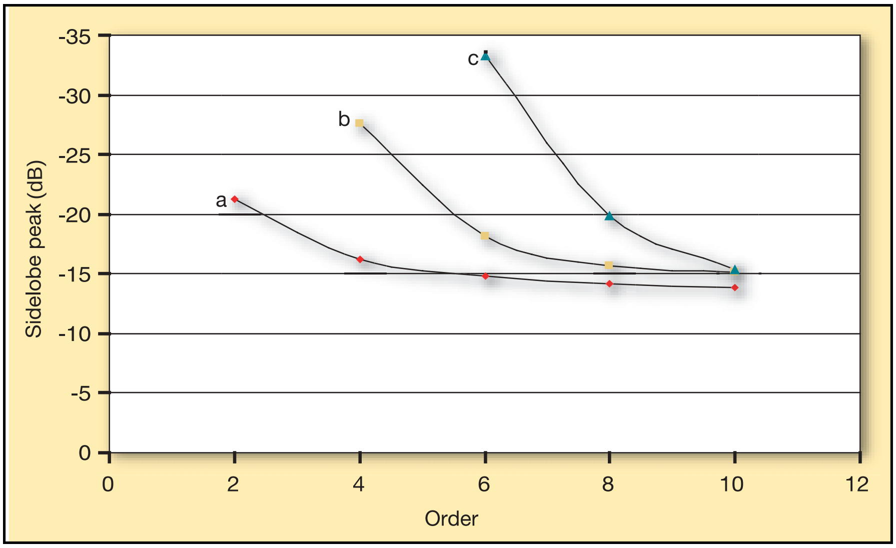

Figure 2. Plot of sidelobe peak vs. order for the Kulkarni window series for asymptotic decays of: a. 12 dB/octave (two-term series), b. 18 dB/octave (three-term series), and c. 24 dB/octave (four-term series).

Behavior of various window functions

The behavior of windows, in terms of their sidelobe peak, asymptotic decay or mainlobe width, is determined by the type of function and the criteria used to determine their coefficients. The Harris windows belong to the family of cosine windows[1-3] and are expressed in time domain as (Eq. 1):

$$w(t) = \sum_{n=0}^{N-1} a_n \cos(2n\pi t / T), \qquad \text{for } |t| \leqslant T/2$$
$$= 0 \qquad \text{for } |t| > T/2$$

where T is the window duration and a_n are real coefficients. There are N terms in the window function. The coefficients, a_n, are determined by imposing either/or of the following conditions:

- The energy in the sidelobes is minimized by forcing the frequency function to vanish at frequencies where the amplitudes of individual terms (in the frequency function) are maximum.
- The terms in the frequency function containing $(1/f), (1/f)^2, \ldots, (1/f)^k$ are forced to zero, so that the window decays as $(1/f)^{k+1}$, giving an asymptotic decay of 6(k+1) dB/octave.

Normally, the first coefficient, a_0, is set to unity to obtain unique coefficients for the function[4]. When all the remaining coefficients in the window are determined using the criterion of minimizing energy in the sidelobes, the window will have lowest sidelobe peaks. But the asymptotic decay remains at 6 dB/octave because the frequency function will have a term containing (1/f). This type of function is not suitable for applications requiring harmonic suppression as the sidelobe decay is not fast enough. To enhance the far-off rejection characteristic, one or more coefficients in the window are to be determined using the second criterion so that sidelobes decay at a faster rate. The three-term and the four-term Harris windows with 18 dB/octave sidelobe decay are constructed in this manner (Table 1).

Assume that all the unknown coefficients in the above function (Eq. 1) are determined using the second criterion only. Then the function will have maximum asymptotic decay, and its coefficients are such that the window is expressed as a powered cosine as given below (Eq. 2):

$$w(t) = \cos^{2(N-1)}(\pi t / T), \qquad \text{for } |t| \leqslant T/2$$
$$= 0 \qquad \text{for } |t| > T/2.$$

This function is called a cosine-powered window and as evident from the Table 1, its asymptotic decay is proportional to the power of the cosine function. However, as the power of the window is increased or in other words, as the number of terms in the window (Eq.1) is increased, the mainlobe width also increases with asymptotic decay. Another set of cosine-powered windows is formed when the function series of the form[3,5], as shown below, are constructed (Eq. 3):

$$w(t) = \sum_{n=0}^{N-1} b_n \cos\left[(2n+1)\pi t / T\right], \qquad \text{for } |t| \leqslant T/2$$
$$= 0 \qquad \text{for } |t| > T/2.$$

The coefficients, b_n, are determined using the criterion of maximum asymptotic decay, and the function gets simplified as given below (Eq. 4):

$$w(t) = \cos^{(2N-1)}(\pi t / T), \qquad \text{for } |t| \leqslant T/2$$
$$= 0 \qquad \text{for } |t| > T/2$$

Window function	No.of terms	Mainlobe width	Peak sidelobe (dB)	Asymptotic decay (dB/octave)
Harris window	3	± 3/T	-64.2	18
	4	± 4/T	-93.3	18
Cosine-powered window				
(Even)	1	± 1.5/T	-23	12
	2	± 2.5/T	-39	24
(Odd)	1	± 2/T	-32	18
	2	± 3/T	-47	30
Kulkarni window (Order)				
2	2	± 1.43/T	-21.3	12
4	2	± 1.232/T	-16.2	12
6	2	± 1.16/T	-14.8	12
8	2	± 1.12/T	-14.2	12
10	2	± 1.098/T	-13.8	12
4	3	± 1.835/T	-27.6	18
6	3	± 1.437/T	-18.1	18
8	3	± 1.303/T	-15.7	18
10	3	± 1.232/T	-15.1	18
6	4	± 2.224/T	-33.3	24
8	4	± 1.622/T	-19.9	24
10	4	± 1.433/T	-15.3	24

Table 1.Characteristics of window functions.

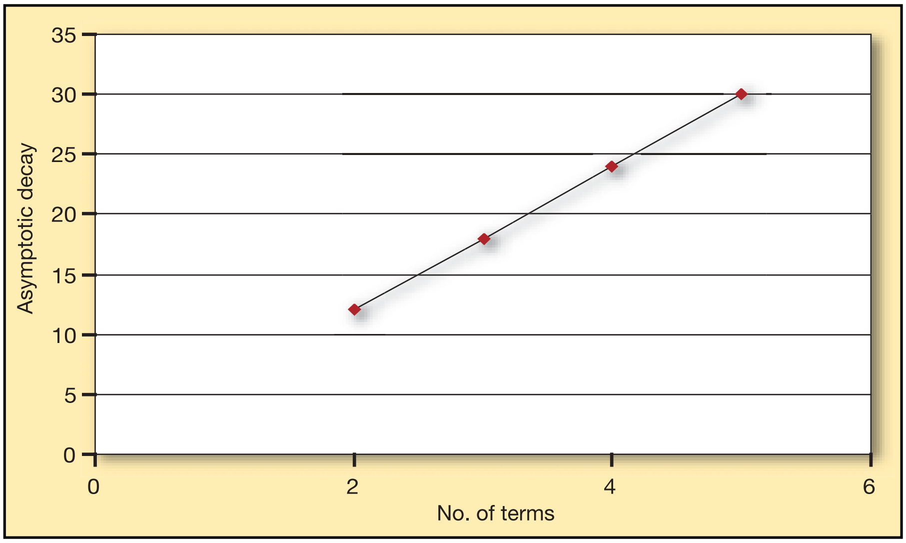

Figure 3. Plot of asymptotic decay vs. number of terms for the Kulkarni window series.

No.of terms	Order	Magnitude
2	2	4
2	4	16
2	6	64
2	8	256
2	10	1024
3	4	16
3	6	128
3	8	768
3	10	4096
4	6	64
4	8	768
4	10	6144

Table 2. Magnitude of largest coefficient in the Kulkarni window series.

The cosine-powered windows defined by Eq. 3 and Eq. 4 can be called as odd and even cosine-powered window functions in the same manner as defined in a paper by Malocha and Bishop[3]. The odd series will have an odd number of kernels, and the even series will have an even number of kernels in their frequency domain functions. These two functions (Eq. 3 and Eq. 4) can be expressed by a single equation as shown below (Eq. 5):

$$w(t) = \cos^k(\pi t / T), \quad \text{for } |t| \leq T/2$$
$$= 0 \quad \text{for } |t| > T/2,$$

where k assumes integer values.

In the recently published papers[6,7], the author has proposed new window functions, which have the narrowest possible mainlobes for any desired asymptotic decay. Theoretically, the mainlobe of these windows can be made as narrow as that of a rectangular window, (±1/T), by increasing the order of the window series. These windows (also known as Kulkarni windows) find application where the signals encountered are extremely narrowband and multitone rejection is essential.

The Kulkarni window series is defined in time domain as[7] (Eq. 6):

$$p_{2N,k}(t) = 1 + \sum_{j=1}^{k} C_{2N-2j+2}(t/T)^{2N-2j+2}, \quad \text{for } |t| \leq T/2$$
$$= 0 \quad \text{for } |t| > T/2,$$

where 2N is the order of the window series, which contains k+1 terms and the coefficients, $C_{2N-2j+2}$, are the real numbers to be determined by imposing the criterion of maximum asymptotic decay so that the terms containing $(1/f),(1/f)^2,\ldots,(1/f)^k$ vanish from the frequency function. The window will then decay as $(1/f)^{k+1}$ giving a sidelobe decay of 6(k+1) dB/octave. The salient features of the Kulkarni window series are given in Table 1 for the cases of two, three and four terms. Note that the asymptotic decay increases with the increase in the number of terms. The mainlobe width can approach ±1/T for any desired sidelobe decay, a feature that is not present in any other window function. Figure 1 shows

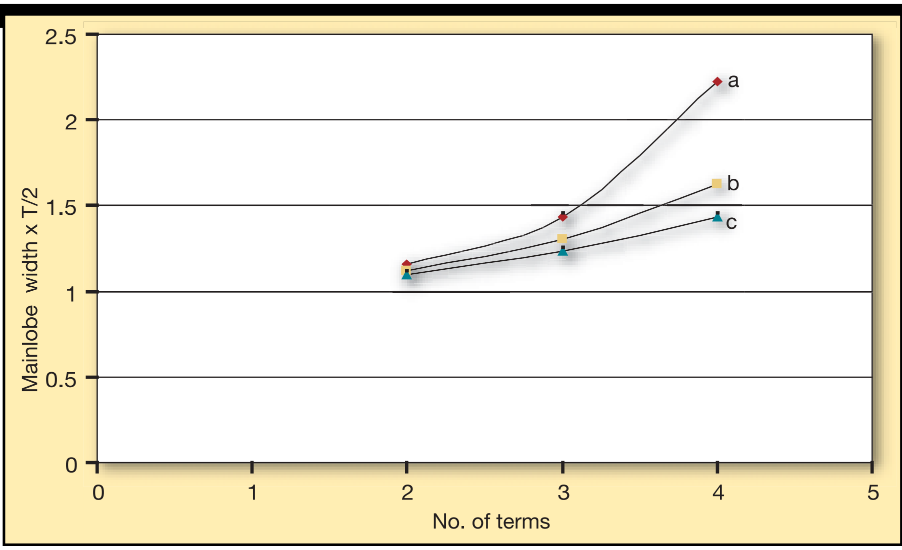

Figure 4. Plot of mainlobe width vs. number of terms for the Kulkarni window series with order of : a. six, b. eight and c. 10.

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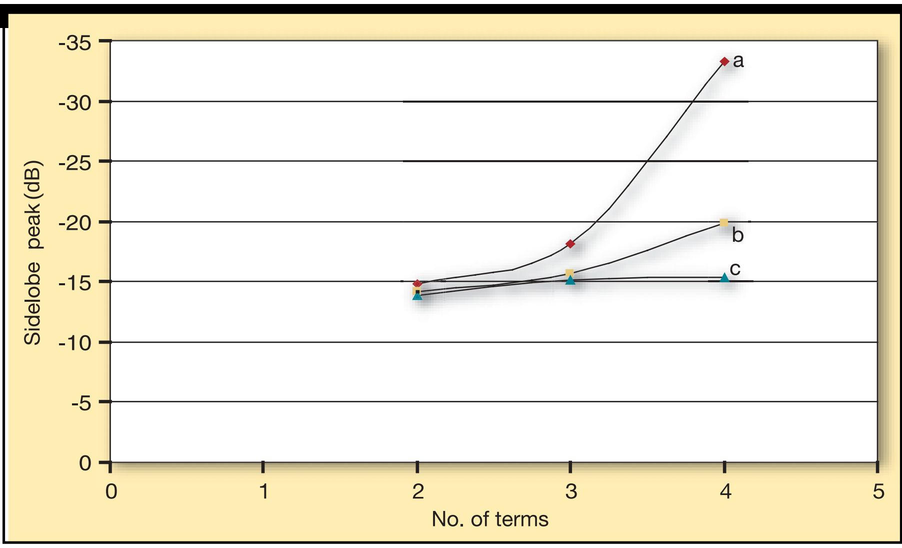

Figure 5. Plot of sidelobe peak vs. number of terms for the Kulkarni window series with order of: a. six, b. eight and c. 10.

how one can reduce the mainlobe width by increasing the order of the series. Theoretically, the mainlobe can be made as small as that of a rectangular window. However, the magnitude of coefficient of highest-order term in the window is the limiting factor. This magnitude increases exponentially as the order of the series is increased (Table 2). A limit on the order of the series is reached when this magnitude becomes so large that the implementing hardware cannot handle it.

Even though there is reduction in mainlobe width as the order of the series is increased, Figure 2 shows that the sidelobe rejection decreases. This is the price one pays for the reduction in mainlobe width. In Figures 1 and 2 the number of terms (or the asymptotic decay) is kept as a parameter while mainlobe width and sidelobe level are plotted against the order of the window. From Figures 3 and 4 we deduce that, while asymptotic decay is determined by the number of terms in the series and not at all dependent on the order, the mainlobe width does depend on both. It decreases with the decrease in the number of terms and with the increase in the order of the series. Sidelobe rejection decreases with the increase in the order (Figure 2). However, it increases with the number of terms in the window function (Figure 5).

To synthesize the Kulkarni window having the narrowest possible mainlobe width for a desired decay of sidelobes, the number of terms and the order (for the window) are to be determined. The number of terms is decided by the specification on the sidelobe decay, and the order is limited by the implementing hardware.

While Harris windows are appropriate when the signals are broadband in nature, cosine-powered windows and Kulkarni windows are suitable for narrowband signals. RFD

Acknowledgments

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